

A Variational Formulation of Distributed Damage in Brittle Materials

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We discuss a constitutive model of damage in the form of distributed fractures or faults with progressively weaken the elasticity of a material. The primary aim of the proposed model is to devise means for the effective generation of sequential-fault microstructures and the computation of the effective behavior of the damaged or faulted material.

The model is consistent with well established cohesive theories of discrete fracture [1], and the constitutive updates are given in a fully variational form [2, 3], by optimizing a suitable incremental work function.

The model describes a particular class of deformations, typical of goematerials, consisting in nested families of equi-spaced cohesive faults bounding otherwise elastic matrix material. The formulation accounts for finite kinematics and is able to include any number of nested fault families. Each family is defined by an orientation \mathbf{N} . The model introduces as a material property the spacing L , which denotes some microstructural feature of the material –such as the aggregate size in concrete– and sets a lower-bound on the distance of fractures.

The approach is based on the multiplicative decomposition of the deformation gradient \mathbf{F} into elastic part \mathbf{F}^e (deformation of the elastic matrix) and inelastic part \mathbf{F}^c (deformation due to the opening displacement Δ across each of the faults).

The corresponding free energy density of the material decomposes into the sum of an elastic strain-energy density per unit volume of the matrix, W^e , and a cohesive energy per unit fault surface, Φ , which properly includes the contribution of each fault family.

Upon load reversal, faults may close and undergo contact, which may include frictional sliding of the blocks. In the context of the variational approach, contact and friction are accounted for by adding to the incremental work function the contribution of the frictional dissipation and including the unilateral contact constraints.

REFERENCES

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